Homework 1

1. **Murder Suspects Puzzle.** Three suspects (Smith, Jones, Williams) each make statements.
   - Smith says: he did not kill Cooper; Cooper was a friend of Jones; Williams disliked Cooper.
   - Jones says: he did not kill Cooper; he did not know Cooper; he was out of town that day.
   - Williams says: he did not kill Cooper; he saw Smith and Jones with Cooper that day; either Smith or Jones killed Cooper.

   Determine the murderer under each assumption:
   (a) Exactly one man is guilty; the two innocent men tell the truth; the guilty man's statements may be true or false.
   (b) Innocent men never lie.

2. **Negating a Conditional.** The negation of the statement "If it rains, the ground is wet" is:
   (a) If the ground is not wet, it does not rain.
   (b) It rains and the ground is not wet.
   (c) If the ground is wet, it does not rain.
   (d) If it rains, the ground is not wet.

3. **Equivalent Sentence.** Which of the sentences is equivalent to the newspaper headline and why?:

   "UK minister refuses to rule out ignoring law preventing no-deal Brexit."

   (a) UK minister does not accept not to rule in not acknowledging law not approving no-deal Brexit.
   (b) UK minister accepts to rule in acknowledging law approving no-deal Brexit.
   (c) UK minister does not accept to rule out ignoring law tolerating Brexit with deal.
   (d) UK minister refuses to rule in acknowledging law approving no-deal Brexit.

4. **Tautology or Contingency?** Let p and q be propositions. Consider:

   $$((q \to p) \wedge \neg q) \to \neg p \quad \text{and} \quad (((\neg q) \to (\neg p)) \wedge p) \to q.$$

   Which option is correct?
   (a) One of the compound propositions is a tautology, the other is a contingency.
   (b) Both compound propositions are contingencies.
   (c) One of the compound propositions is a contradiction, the other is a contingency.
   (d) One of the compound propositions is a tautology, the other is a contradiction.

5. **Equivalence Proof.** Using only the axioms and theorems established in the notes (not truth tables), prove the following equivalence:

   $$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r).$$

   Each step must cite the specific axiom or theorem used.

6. **Heron's Method Revisited.** Starting from $x = 2$ and initial guess $g = 1$, manually compute three iterations of Heron's method using only Python assignment statements and the convergence test $|g^2 - x| < 0.001$. Write your computation as a sequence of reassignments to `g`, printing both `g` and the value of `g ** 2 - x` after each step.

```
x = 2
g = 1.0
# Iteration 1
g = 0.5 * (g + x / g)
print(g, g ** 2 - x)
# Continue for iterations 2 and 3...
```

   Does the algorithm terminate within three iterations? After computing all three iterations, calculate the error $e_n = g_n^2 - x$ at each step. Compute the ratios e_2/e_1^2 and e_3/e_2^2. What pattern do you observe in how the error shrinks from one iteration to the next?

7. **DNF Construction.** Find the Disjunctive Normal Form (DNF) of:

$$(p \to q) \land (q \to r)$$

using the truth table method. Simplify the result as far as possible, citing each equivalence used. Verify your simplified DNF against the original formula for at least four distinct assignments using Python.

8. **CNF Identification.** Show that the conjunctive normal form (CNF) of $p \to (q \oplus r)$ is:

$$(\neg p \lor q \lor r) \land (\neg p \lor \neg q \lor \neg r).$$

You may use either algebraic manipulation or the truth table method, but you must show your working.

9. **Formula Classification.** The compound proposition

$$((\neg p \land q) \to (r \oplus q)) \lor (\neg s \leftrightarrow p)$$

involves four propositional variables. Without constructing the full 16-row truth table, determine whether this formula is a tautology, a contingency, or a contradiction. Justify your answer. *Hint:* a single well-chosen assignment can rule out two of the three possibilities.

10. **Finitude of Beliefs.** Determine the truth value of the sentence:

"You have finitely many beliefs."

The "you" above refers to the reader of these notes.