Recitation 1

1. **Converse Practice.** For each implication below, write its converse.
   (a) If it rains, then the ground is wet.
   (b) If n is divisible by 4, then n is even.
   (c) If a function is differentiable, then it is continuous.

2. **Equivalences Without Truth Tables.** Show the following without using truth tables:
   (a) $\neg p \leftrightarrow q \equiv p \leftrightarrow \neg q$.
   (b) $p \oplus (q \wedge u) \not\equiv (p \oplus q) \wedge (p \oplus u)$.
   (c) $p \oplus q \equiv (p \vee q) \wedge (\neg p \vee \neg q)$.
   (d) $\neg(p \oplus q) \equiv (\neg p) \oplus q$.
   (e) $p \leftrightarrow q \equiv \neg(p \oplus q)$.

3. **State Tracing.** Determine the final values of `x`, `y`, and `z` after the following sequence of statements, *without* running the code. Then verify by running it.

```
x, y = 3, 7
z = x + y
x, y = y, x + z
z = z % y
```

   What are the final values of `x`, `y`, and `z`? At which step does multiple assignment become critical to getting the correct answer?

4. **Targeted Construction.** Construct a compound proposition in variables p, q, u that is true iff:
   (a) p is true, q is false, and u is false.
   (b) Exactly one of p, q, u is true.
   (c) At least two of p, q, u are true.

5. **Knights and Knaves.** On an island, Knights always tell the truth and Knaves always lie. Person A says:

   "If I am a Knight, then B is a Knight."

   Determine what A and B can be.

6. **Type Prediction.** For each expression below, predict (a) the value and (b) the type of the result. Do not run the code first.

```
5 / 2 == 5 // 2 # (i)

type(True + True + False) # (ii)

not (7 % 3 == 1) or (2 ** 3 == 8) # (iii)

10 // 3 * 3 + 10 % 3 == 10 # (iv)
```

   For expression (iv), explain why the result holds for any positive integers in place of `10` and `3`, connecting your reasoning to Problem 6 from the notes.

7. **Chat Room Puzzle.** Five friends have access to a chat room. Is it possible to determine who is chatting if:
   (a) Either Kevin or Heather, or both, are chatting.
   (b) Either Randy or Vijay, but not both, are chatting.
   (c) If Abbey is chatting, so is Randy.
   (d) Vijay and Kevin are either both chatting or neither is.
   (e) If Heather is chatting, then so are Abbey and Kevin.

   Explain your reasoning.