

# Homework

## 1. Percentage Rate of Change.

- (a) Determine the percentage rate of change of  $f(t) = t^2$  at  $t = 10$  and at  $t = 50$ .
- (b) The annual sales  $S$ , in pounds, of a company are modelled by

$$S(t) = 50000e^{\sqrt{t}},$$

where  $t$  is the number of years after a fixed reference date. Use a logarithmic derivative to determine the percentage rate of growth of sales at  $t = 4$ .

## 2. Elasticity of Demand.

For a demand function  $q = f(p)$ , use

$$E(p) = -\frac{pf'(p)}{f(p)}.$$

Find  $E(p)$  and decide whether demand is elastic, inelastic, or unit-elastic at the indicated price.

- (a)  $q = 700 - 5p$ , at  $p = 80$ .
- (b)  $q = 400(116 - p^2)$ , at  $p = 6$ .

## 3. Commuter Train Revenue.

Currently 1800 people ride a certain commuter train each day and pay \$4 for a ticket. The number of people  $q$  willing to ride the train at price  $p$  is

$$q = 600(5 - \sqrt{p}).$$

The railroad would like to increase its revenue.

- (a) Check that the model agrees with the current data at  $p = 4$ .
- (b) Is demand elastic or inelastic at  $p = 4$ ?
- (c) Should the price of a ticket be raised or lowered if the goal is to increase revenue? Justify using elasticity.

## 4. Bounded Growth.

Consider

$$f(x) = 5(1 - e^{-2x}), \quad x \geq 0.$$

- (a) Show that  $f$  is increasing and concave down for all  $x \geq 0$ .
- (b) Explain why  $f(x)$  approaches 5 as  $x$  gets large.
- (c) Sketch the graph of  $f$  on  $x \geq 0$ , labelling the horizontal asymptote.
- (d) If  $y = 2(1 - e^{-x})$ , compute  $y'$  and show that  $y' = 2 - y$ .

## 5. Spread of News.

A news item spreads by word of mouth to a potential audience of 10000 people. After  $t$  days,

$$f(t) = \frac{10000}{1 + 50e^{-0.4t}}$$

people have heard the news.

- (a) Approximately how many people have heard the news after 7 days?
- (b) At approximately what rate is the news spreading after 14 days?
- (c) Approximately when have 7000 people heard the news?

- (d) Approximately when is the news spreading at 600 people per day?
- (e) When is the news spreading at the greatest rate?
- (f) Verify that  $f$  satisfies a differential equation of the form

$$f'(t) = c f(t)(10000 - f(t))$$

for a constant  $c$ , and find  $c$ .

- (g) At what rate is the news spreading when half the potential audience has heard the news?