

Homework

1. Implicit Derivative at a Point. Suppose x and y are related by

$$x^2 + y^2 = 1.$$

Find $\frac{dy}{dx}$ at the point

$$x = y = \frac{\sqrt{2}}{2}.$$

2. Implicit Differentiation. For each equation, find $\frac{dy}{dx}$ in terms of x and y .

(a) $x^2 - y^2 = 1$.

(b) $y^5 - 3x^2 = x$.

(c) $y^4 - x^4 = y^2 - x^2$.

(d) $2x^3 + y = 2y^3 + x$.

(e) $xy = 5$.

(f) $x(y + 2)^5 = 8$.

3. More Implicit Curves. For each equation, find $\frac{dy}{dx}$, then identify any points where the tangent line is horizontal.

(a) $x^4 + y^4 = 1$.

(b) $3xy - 3x^2 = 4$.

(c) $x^2 + 2xy = y^3$.

4. Demand Equation. Suppose that x and y represent the amounts of two basic inputs for a production process and that

$$10x^{1/2}y^{1/2} = 600.$$

Find $\frac{dy}{dx}$ when $x = 50$ and $y = 72$.

5. Price and Weekly Sales. Suppose that the price p , in dollars, and the weekly sales x , in thousands of units, satisfy

$$2p^3 + x^2 = 4500.$$

Determine the rate at which sales are changing when $x = 50$, $p = 10$, and the price is falling at the rate of 0.50 dollars per week.

6. Advertising Revenue. The monthly advertising revenue A , in thousands of dollars, and the monthly circulation x , in thousands of copies sold, are related approximately by

$$A = 6\sqrt{x^2 - 400}, \quad x \geq 20.$$

At what rate is advertising revenue changing when the current circulation is 25 thousand copies and circulation is growing at 2 thousand copies per month?

Hint. Use

$$\frac{dA}{dt} = \frac{dA}{dx} \frac{dx}{dt}.$$

7. Function Rules. Let f , g , and h be differentiable functions.

(a) Find a formula for $\frac{d}{dx}(f(g(h(x))))$.

(b) Find a formula for $\frac{d}{dx}(f(x)^2)$ in terms of $f(x)$ and $f'(x)$.

(c) Using the product rule, not the quotient rule, find a formula for $\frac{d}{dx}\left(\frac{1}{f(x)}\right)$, assuming $f(x) \neq 0$.

8. Recovering an Unknown Derivative. Suppose f and g are differentiable functions satisfying

$$f(g(x)) = \frac{1}{x}, \quad g(1) = 0, \quad f'(0) = 1.$$

Find $g'(1)$.

9. The Square Root from the Definition. The Power Rule has been used for rational exponents, but this problem asks you to verify one important case directly. For $x > 0$, prove from the definition of the derivative that

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}.$$

Hint. Rationalise

$$\frac{\sqrt{x+h} - \sqrt{x}}{h}$$

before taking the limit as $h \rightarrow 0$.

10. Several Rules at Once. Differentiate each function. State where the derivative is defined.

(a) $y = 6x^2(x-1)^3$.

(b) $y = 2(x^3 - 1)(3x^2 + 1)^4$.

(c) $y = \left(\frac{3x+1}{4x-1}\right)^3$.

(d) $y = \frac{x+3}{(2x+1)^2}$.

11. Exponential Algebra Preview. Use only the laws of exponents and injectivity of exponential functions to solve.

(a) $5^{2x} = 5^x$.

(b) $(2.5)^{2x+1} = (2.5)^5$.

(c) $2^x - \frac{8}{2^x} = 0$.

(d) $2^{2x} - 12 \cdot 2^x + 27 = 0$.