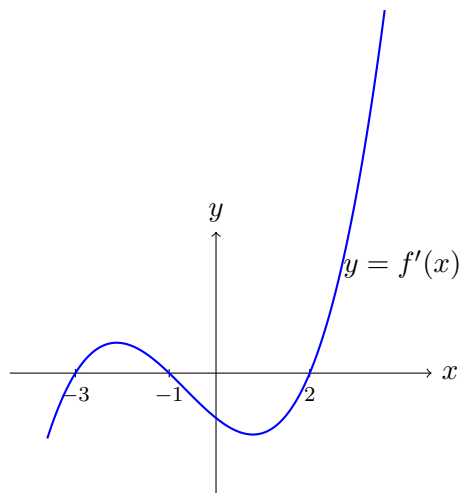


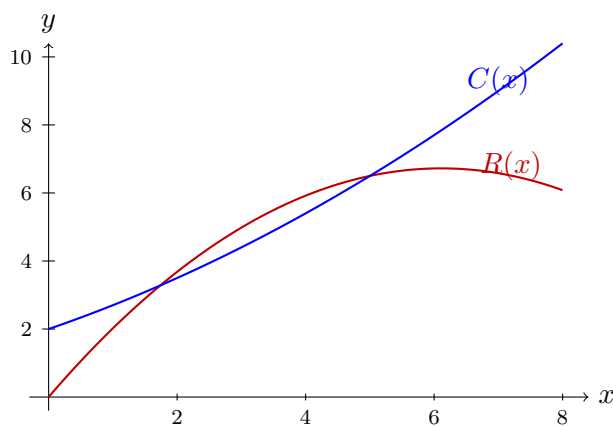
## Homework

1. **Reading a Derivative Graph.** The graph below is the graph of  $y = f'(x)$ , not the graph of  $f(x)$ .



- On which intervals is  $f$  increasing? On which intervals is  $f$  decreasing?
- At which  $x$ -values does  $f$  have relative extrema? Classify each one as a relative maximum or relative minimum.
- On which intervals does the slope of  $f$  appear to be increasing? On which intervals does it appear to be decreasing?
- Sketch one possible graph of  $f$ , matching the signs of  $f'$  and the extrema forced by them.

2. **Cost, Revenue, and Profit from Graphs.** The two curves below show total revenue  $R(x)$  and total cost  $C(x)$ , in thousands of pounds, when  $x$  thousand units are produced and sold.



- Estimate the break-even production levels from the graph.
- On which interval does the company appear to make a positive profit?
- Around which production level does profit appear to be largest?
- Explain, using the slopes of the two curves, why the maximum profit should occur where the tangent slopes of  $R$  and  $C$  are equal.

3. **A Full Polynomial Sketch.** Let

$$f(x) = x^3 - 6x^2 + 9x.$$

- (a) Find  $f'(x)$  and  $f''(x)$ .
- (b) Find all critical numbers and classify the corresponding points using the first-derivative test.
- (c) Find the inflection point.
- (d) Find the  $y$ -intercept and the  $x$ -intercepts.
- (e) Sketch the curve, marking the relative extrema and the inflection point.

**4. A Curve with No Relative Extrema.** Let

$$g(x) = \frac{1}{3}x^3 - 2x^2 + 5x - 1.$$

- (a) Show that  $g'(x)$  is never zero.
- (b) Decide whether  $g$  is increasing or decreasing for every real  $x$ .
- (c) Find the inflection point and the concavity intervals.
- (d) Sketch the graph, including the  $y$ -intercept.

**5. Rational Curve Sketching.** Let

$$h(x) = x + \frac{4}{x}, \quad x \neq 0.$$

- (a) Find  $h'(x)$  and  $h''(x)$ .
- (b) Find all relative extrema and classify them.
- (c) Find all vertical and oblique asymptotes.
- (d) Determine the concavity on each side of  $x = 0$ .
- (e) Sketch the graph.

**6. Largest Rectangle Under a Parabola.** A rectangle is placed under the parabola

$$y = 12 - x^2$$

with its base on the  $x$ -axis and its upper two corners on the parabola. Assume the rectangle is symmetric about the  $y$ -axis.

- (a) If the right upper corner is  $(x, 12 - x^2)$ , write the area  $A(x)$  of the rectangle.
- (b) State the natural domain of  $x$ .
- (c) Find the dimensions of the rectangle with maximum area.
- (d) Find the maximum area.

**7. Open-Top Box.** A square sheet of cardboard has side length 18 cm. Equal squares of side length  $x$  are cut from the four corners, and the sides are folded upward to form an open-top box.

- (a) Write the volume  $V(x)$  as a function of  $x$ .
- (b) State the natural domain of  $x$ .
- (c) Find the value of  $x$  that gives the maximum volume.
- (d) Find the maximum volume.

**8. Economic Order Quantity.** A shop sells 5000 notebooks per year. Each order costs £36 to place, and holding one notebook in inventory for a year costs £2. Assume orders arrive instantly and inventory decreases linearly between orders.

- (a) If  $x$  notebooks are ordered at a time, write the annual ordering cost.
- (b) Write the annual holding cost.
- (c) Write the total annual inventory cost  $C(x)$ .
- (d) Find the order size  $x$  that minimizes  $C(x)$ .
- (e) At the minimizing order size, compare annual ordering cost and annual holding cost.

**9. Demand, Revenue, and Price.** A monopolist's demand equation is linear. At price £18, weekly demand is 400 units; at price £14, weekly demand is 600 units.

- (a) Find the linear demand equation  $p = f(x)$ , where  $x$  is weekly demand.
- (b) Write the revenue function  $R(x) = xf(x)$ .
- (c) Find the production level and price that maximize weekly revenue.
- (d) Find the maximum weekly revenue.

**10. Profit and an Excise Tax.** A monopolist faces the demand equation

$$p = 80 - 0.02x$$

and has cost function

$$C(x) = 30x + 4000.$$

- (a) Find the profit function  $P(x)$ .
- (b) Find the production level and price that maximize profit.
- (c) The government imposes an excise tax of £8 per unit. Find the new profit-maximizing production level and price.
- (d) How much of the £8 tax is passed on to consumers through the higher optimal price?

**11. A Parameter Family of Cubics.** Let  $a > 0$  and

$$f_a(x) = x^3 - 3a^2x.$$

- (a) Find the critical numbers in terms of  $a$ .
- (b) Classify the corresponding relative extreme points.
- (c) Find the inflection point.
- (d) Compute the vertical distance between the relative maximum point and the relative minimum point.
- (e) Explain how the sketch changes as  $a$  increases.