1 Homework

1. **Derivative Rules for Powers.** Differentiate each function. Write radical functions using powers first, then simplify where reasonable.

$$-2x, \qquad x^{-2}, \qquad \frac{1}{x^3}, \qquad \frac{1}{\sqrt[5]{x}}, \qquad \sqrt{x^2}, \qquad \pi.$$

   For $\sqrt{x^2}$, state carefully where the derivative exists.

2. **Slopes at Specified Inputs.** Find the slope of the curve at the stated input.
   (a) $f(x) = x^5$ at $x = \frac{3}{2}$.
   (b) $f(x) = x^{1/3}$ at $x = 8$.
   (c) $y = x^5$ at $x = \frac{1}{3}$.
   (d) $f(x) = x^2$ at $x = -\frac{1}{2}$.

3. **Tangent Lines.** Find the equation of the tangent line in slope-intercept form.
   (a) $y = x^2$ at $x = \frac{3}{2}$.
   (b) $y = x^3 + 3x - 8$ at the point $(2, 6)$.
   (c) $y = x^{-2}$ at $x = 1$. Also compute $f(1)$ and $f'(1)$.

4. **A Perpendicular Tangent Question.** Is there any point on the graph of $y = x^3$ where the tangent line is perpendicular to the line $y = x$? Justify your answer.

5. **Difference Quotients as Limits.** Evaluate each limit.

$$\lim_{h\to 0} \frac{(1+h)^2 - 1}{h}, \qquad \lim_{h\to 0} \frac{(2+h)^3 - 8}{h},$$

$$\lim_{h\to 0} \frac{\sqrt{9+h} - 3}{h}, \qquad \lim_{h\to 0} \frac{(1+h)^{-1/2} - 1}{h}.$$

6. **Combining Derivatives.** Suppose

$$f(5) = 2, \qquad f'(5) = 3, \qquad g(5) = 4, \qquad g'(5) = 1,$$

   and define $h(x) = 3f(x) + 2g(x)$. Find $h(5)$ and $h'(5)$.

7. **Manufacturing Cost.** Let $C(x)$ be the cost, in dollars, of manufacturing x bicycles per day in a certain factory. Interpret each statement in ordinary language:

$$C(50) = 5000, \qquad C'(50) = 45.$$

   Then estimate the cost of manufacturing 51 bicycles per day.

8. **Estimating from a Derivative.** Suppose $f(25) = 10$ and $f'(25) = -2$. Use the tangent-line approximation near $x = 25$ to estimate

$$f(27), \qquad f(26), \qquad f(25.25), \qquad f(24), \qquad f(23.5).$$

9. **Continuity and Differentiability at a Joining Point.** For each function, decide whether it is continuous at the joining point and whether it is differentiable there. Give a reason in each case.
   (a)

$$f(x) = \begin{cases} x+2, & -1 \le x \le 1, \\ 3x, & 1 < x \le 5. \end{cases}$$

(b)

$$f(x) = \begin{cases} x^3, & 0 \le x < 1, \\ x, & 1 \le x \le 2. \end{cases}$$

(c)

$$f(x) = \begin{cases} x - 1, & 0 \le x < 1, \\ 1, & x = 1, \\ 2x - 2, & x > 1. \end{cases}$$

10. A Removable Break. Let

$$f(x) = \frac{x^3 - 5x^2 + 4}{x^2}, \qquad x \neq 0.$$

(a) Rewrite $f(x)$ as a sum of powers of x.
(b) Find $f'(x)$ for $x \neq 0$.
(c) Explain why the function is not continuous at $x = 0$, even if the algebraic expression can be written using powers.

11. Progressive Tax Function. A single taxpayer pays tax according to the following brackets.

Amount over	But not over	Tax rate
\$0	\$27,050	15%
\$27,050	\$65,550	27.5%
\$65,550	\$136,750	30.5%

Let x be taxable income and $T(x)$ be the tax owed, for $0 \le x \le 136{,}750$.

(a) Find a piecewise formula for $T(x)$.
(b) Sketch the graph of $T(x)$ on this interval.
(c) Is T continuous at the bracket boundaries? Explain.
(d) What is the meaning of the slope of each linear piece?
(e) Find the maximum amount of tax paid on the portion of income in the second bracket. Express your answer as a difference of two values of T.

12. Rates of Motion from Graph Shape. A car travels along a straight road, and $s(t)$ denotes its signed distance from its starting point at time t. Match each behaviour with the graph shape that best represents it.

Graph shape	Description
A	increasing straight line
B	horizontal line
C	decreasing straight line
D	increasing curve getting steeper
E	increasing curve flattening out

Behaviours:

(a) The car travels at a steady speed.
(b) The car is stopped.
(c) The car is backing up.
(d) The car is accelerating forward.
(e) The car is decelerating while still moving forward.

13. Secant Slopes Approaching a Tangent. Let f be a function defined for every real number. Suppose that for every real number a and every nonzero real number h,

$$\frac{f(a+h)-f(a)}{h}=3a^2+3ah+h^2+2.$$

This formula gives the slope of the secant line through the points $(a, f(a))$ and $(a+h, f(a+h))$.

(a) Fix $a = 2$. Compute the secant slope for $h = 1,\ \frac{1}{2},\ \frac{1}{10},\ -\frac{1}{10}$.

(b) For a general fixed a, what number do the secant slopes approach as $h \to 0$?

(c) Explain why the answer in (b) should be the slope of the tangent line to the graph of f at $x = a$.

(d) Since $f(0) = b$ for some constant b, put $a = 0$ and $h = x$ to show that, for $x \neq 0$,

$$f(x) = x^3 + 2x + b.$$

(e) Verify directly from $f(x) = x^3 + 2x + b$ that the tangent slope found in (b) agrees with the Power Rule.