

1 Homework

- 1. Factoring and Intercepts.** Factor each polynomial completely over the real numbers. Then use the factorisation to find the x -intercepts of the corresponding graph $y = p(x)$.

$$x^2 + 8x + 15, \quad x^2 - 10x + 16, \quad x^2 - 16, \quad x^2 - 1.$$

- 2. Quadratic Formula and Real Roots.** Solve each equation over the real numbers. If an equation has no real solution, explain why.

- (a) $5x^2 - 4x - 1 = 0$.
- (b) $x^2 - 4x + 5 = 0$.
- (c) $2x^2 + 3x - 2 = 0$.

- 3. Intersections of Curves.** Find all points of intersection of each pair of curves.

- (a) $y = 2x^2 - 5x - 6$ and $y = 3x + 4$.
- (b) $y = x^2 - 10x + 9$ and $y = x - 9$.
- (c) $y = x^2 - 4x + 4$ and $y = 12 + 2x - x^2$.
- (d) $y = 3x^2 + 9$ and $y = 2x^2 - 5x + 3$.

- 4. A Composition Model.** The function $g(x) = 8x + 1$ converts a British men's hat size x to a French size. The function $f(x) = \frac{1}{8}x$ converts a French size x to a U.S. size.

- (a) Determine $h(x) = f(g(x))$ and simplify.
- (b) Interpret $h(x)$ in one sentence.
- (c) Use h to convert the British sizes $6\frac{1}{2}$, 7 , and $7\frac{3}{8}$ to U.S. sizes.
- (d) Find the British hat size corresponding to U.S. size $7\frac{1}{2}$.

- 5. Combining Three Functions.** Let

$$f(x) = x^6, \quad g(x) = \frac{x}{1-x}, \quad h(x) = x^3 - 5x^2 + 1.$$

- (a) Compute $f(g(x))$ and state its natural domain.
- (b) Compute $g(f(x))$ and state its natural domain.
- (c) Compute $g(h(x))$ and state its natural domain.
- (d) Compute $h(x+t)$ and expand the answer as a polynomial in x and t .

- 6. Difference Quotient of a Cubic.** Let $p(x) = x^3 - 5x^2 + 1$.

- (a) Compute $p(x+t) - p(x)$.
- (b) Assuming $t \neq 0$, simplify the difference quotient

$$\frac{p(x+t) - p(x)}{t}.$$

- (c) Evaluate the simplified expression at $t = 0$. This is not yet a derivative calculation; it is only the algebraic expression that will survive when the derivative is introduced.

- 7. Power Notation and Radicals.** Simplify each expression as far as possible. State any restriction on x needed for the expression to make sense over the real numbers.

$$\frac{x^{-4}}{x^3}, \quad (9x)^{-1/2}, \quad \frac{2x}{\sqrt{x}}, \quad \frac{x^2 - 16}{x^2 - 1}.$$

- 8. Factoring Radical Expressions.** Assume $x > 0$ and $y > 0$. Fill in the missing factor in each identity.

$$\sqrt{x} - \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x}}(\quad),$$

$$2x^{2/3} - x^{-1/3} = x^{-1/3}(\quad),$$

$$x^{-1/4} + 6x^{1/4} = x^{-1/4}(\quad),$$

$$\sqrt{\frac{x}{y}} - \sqrt{\frac{y}{x}} = \sqrt{xy}(\quad).$$

- 9. Square-Root Laws.** Let $a, b \geq 0$ and let $c > 0$.

- Explain why $\sqrt{a}\sqrt{b} = \sqrt{ab}$.
- Explain why $\sqrt{a/c} = \sqrt{a}/\sqrt{c}$.
- Give one numerical example showing that $\sqrt{a+b}$ is not usually equal to $\sqrt{a} + \sqrt{b}$.

- 10. A Rational Model Near a Boundary.** For $0 \leq x \leq 100$, let

$$C(x) = \frac{50x}{105 - x},$$

where $C(x)$ is measured in millions of dollars.

- Compute $C(70)$, $C(75)$, $C(95)$, and $C(100)$.
- Compare the cost increase from 70% to 75% with the cost increase from 95% to 100%.
- In one sentence, explain what this comparison says about the graph of C near $x = 100$.