Exam

1. Review: Limits and Tangent Slopes. Let

$$f(x) = \frac{x^2 - 1}{x - 1}, \qquad x \neq 1.$$

(a) Compute $\lim_{x \to 1} f(x)$.

(b) Define $g(1)$ so that the extended function $g(x)$ is continuous at $x = 1$.

(c) For $h(x) = x^2 - 4x + 1$, compute $h'(a)$ from the limit definition.

(d) Find the tangent line to $y = h(x)$ at $x = 3$.

2. Product, Quotient, and Chain Rules. Differentiate and simplify where reasonable.

(a) $y = (x^2 + 1)^4(3x - 2)$.

(b) $y = \dfrac{x^2 + 3x - 1}{x}$.

(c) $y = \left(\dfrac{4x - 1}{3x + 2}\right)^3$.

(d) $y = x^2 e^{-3x}$.

3. Implicit Differentiation. The curve

$$x^2 - 3xy + y^2 = 7$$

is given implicitly.

(a) Find $\dfrac{dy}{dx}$ in terms of x and y.

(b) Show that $(2, -1)$ lies on the curve.

(c) Find the tangent line at $(2, -1)$ in the form $y = mx + b$.

4. Related Rates. A 13-foot ladder leans against a vertical wall. The bottom slides away from the wall at 0.5 feet per second.

(a) Find the rate at which the top is sliding down the wall when the bottom is 5 feet from the wall.

(b) The triangle enclosed by the wall, the ground, and the ladder has area $A = \frac{1}{2}xy$. Find $\dfrac{dA}{dt}$ at the same instant.

5. Exponential and Logarithmic Differentiation. Differentiate each function.

(a) $y = e^{2x}(x^2 + 1)$.

(b) $y = \ln(x^3 + 4x + 1)$, on its natural domain.

(c) $y = (x^2 + 1)^x$, for $x > 0$, using logarithmic differentiation.

6. Continuous Compound Interest. An account contains \$4000 at time 0 and earns 3.5% interest compounded continuously.

(a) Write $A(t)$, the balance after t years.

(b) When does the balance first reach \$6000?

(c) How fast is the balance growing at that moment?

7. **Elasticity and Revenue.** A demand curve is

$$q = 900 - 6p.$$

(a) Find $E(p)$.

(b) Find the unit-elastic price.

(c) At $p = 100$, is demand elastic or inelastic?

(d) At $p = 100$, would a small price increase raise or lower revenue?

8. **Bounded Growth and News Spread.** A broadcast reaches a town of 20000 people according to

$$N(t) = 20000(1 - e^{-0.3t}),$$

where t is measured in hours.

(a) Find $N'(t)$.

(b) Verify that $N'(t) = 0.3(20000 - N(t))$.

(c) When have 15000 people heard the broadcast?

(d) At what rate is the news spreading at that time?

9. **Newton's Method.** Use Newton's method to solve

$$\ln x = \frac{1}{x}, \qquad x > 0,$$

starting from $x_0 = 2$.

(a) Write the equation as $f(x) = 0$.

(b) Compute $f'(x)$.

(c) Compute x_1, x_2, x_3 to four decimal places.

10. **Curve Sketching and Optimization Review.** Let

$$C(x) = 0.01x^3 - 0.9x^2 + 30x + 80, \qquad x \geq 0.$$

(a) Compute $C'(x)$ and $C''(x)$.

(b) Find the critical numbers of C on $x \geq 0$.

(c) Determine where C is increasing and decreasing.

(d) Determine where C is concave up and concave down.

(e) Interpret $C'(30)$ as a marginal cost.