Exam

1. Function Algebra and Composition. Let

$$f(x) = 7x^2 + 5x - 3, \qquad g(x) = \frac{2}{x}, \qquad h(x) = 7, \qquad j(x) = 5\sqrt{x}.$$

Give a simplified expression for each of the following, stating any natural domain restrictions that are forced by the formula.

(a) $f(x) + h(x) - j(x)$.

(b) $f(g(x))$.

(c) $g(f(x))$.

(d) $\dfrac{j(x)}{g(x)}$.

(e) $\dfrac{f(x+t) - f(x)}{t}$, where $t \neq 0$.

2. Derivative from the Definition. Find the derivative of

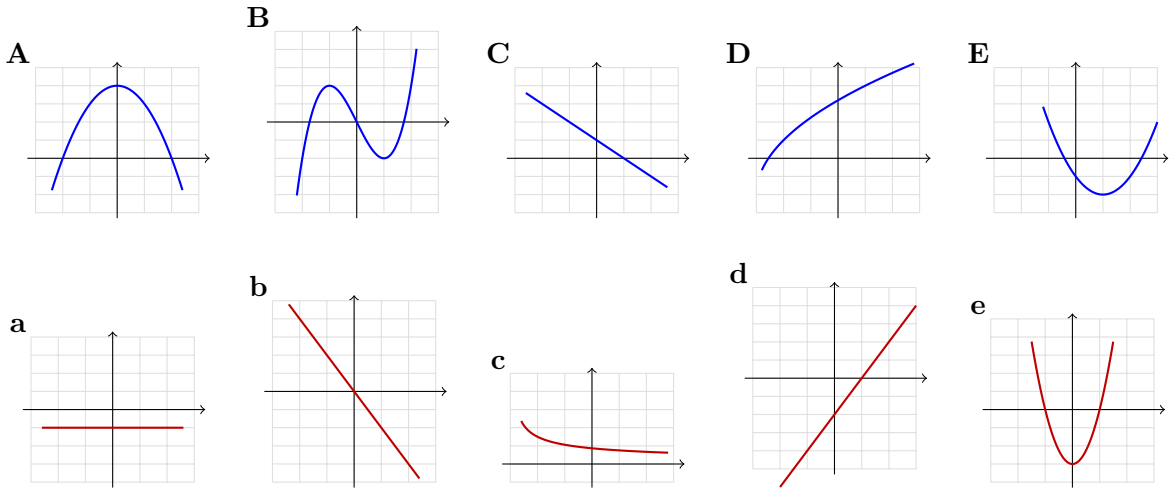
$$f(x) = x^2 - 4x + 1$$

directly from the definition

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

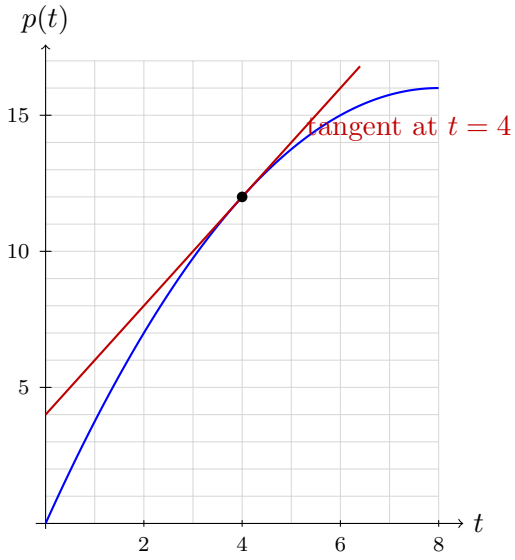
Then find the equation of the tangent line to the graph at $x = 3$.

3. Matching Functions to Their Derivatives. Match the five graphs *A*-*E* with the five graphs *a*-*e* of their derivatives. Explain your reasoning for two of the matches.



4. Motion from a Graph. Curtis runs along a straight path. His distance travelled $p(t)$, in meters,

after t seconds is shown below, along with the tangent line at $t = 4$.



(a) How far has Curtis gone after 6 seconds?

(b) What is his average velocity over the first 4 seconds?

(c) What is his instantaneous velocity after 4 seconds?

(d) Is he going faster when $t = 2$ or when $t = 5$? Explain from the graph.

(e) Is $p''(3)$ positive or negative? Explain.

5. Horizontal Tangents. Let

$$f(x) = \frac{2}{3}x^3 + \frac{1}{2}x^2 - x - 1.$$

Find all points on the graph at which the tangent line has slope 0. Classify each point as a relative maximum or relative minimum, and give the equation of one of the horizontal tangent lines.

6. Cubic Sketch. Let

$$f(x) = x^3 - 6x^2 + 9x + 5.$$

Sketch its graph. Label all relative maxima, relative minima, and inflection points. Also give the intervals on which the function is increasing and decreasing.

7. Open-Top Box. A rectangle of cardboard is 20 cm by 12 cm. Squares of side length x are cut from the four corners, and the sides are folded up to make an open-top box.

(a) Write the volume $V(x)$ as a function of x.

(b) State the natural domain of x.

(c) Find the value of x that maximizes the volume.

(d) Find the maximum volume.

8. Profit and Tax. A monopolist faces the demand equation

$$p = 120 - 0.04x$$

and has cost function

$$C(x) = 40x + 12{,}000.$$

(a) Find the profit function.

(b) Find the production level and price that maximize profit.

(c) A tax of £16 per unit is imposed. Find the new profit-maximizing production level and price.

(d) How much of the tax is passed on to the consumer through the higher price?

9. Rational Curve Sketching. Let

$$r(x) = x + \frac{9}{x}, \qquad x \neq 0.$$

Find the vertical and oblique asymptotes, all relative extrema, and the concavity intervals. Then sketch the graph.

10. Closest Point on a Parabola. Find the point or points on the parabola $y = x^2$ closest to the point $(0, 3)$.

(a) Write the square of the distance from (x, x^2) to $(0, 3)$ as a function of x.

(b) Minimize this function.

(c) Give the closest point or points and the minimum distance.