Exercises

1. Savings Account. Let

$$A(t) = 5000e^{0.04t}$$

be the balance in a savings account after t years.

(a) How much money was originally deposited?

(b) What is the continuously compounded interest rate?

(c) How much money will be in the account after 10 years?

(d) What differential equation is satisfied by $y = A(t)$?

(e) Use parts (c) and (d) to determine how fast the balance is growing after 10 years.

(f) How large will the balance be when it is growing at the rate of $280 per year?

2. A Second Savings Account. Four thousand dollars is deposited in a savings account at 3.5% interest compounded continuously.

(a) Find a formula for $A(t)$, the balance after t years.

(b) What differential equation is satisfied by $A(t)$?

(c) How much money will be in the account after 2 years?

(d) When will the balance reach $5000?

(e) How fast is the balance growing when it reaches $5000?

3. Recovering a Continuous Rate. An investment triples in 15 years under continuous compounding. What continuously compounded annual interest rate does the investment earn?

4. Real Estate Investment. In a certain town, property values tripled from 1980 to 1995. Assume an exponential model

$$V(t) = V_0e^{rt},$$

where t is the number of years after 1980. If the same trend continues, in what year will property values be five times their 1980 level?

5. Newton's Method at the End of the Course. Use Newton's method to solve

$$xe^x = 2$$

starting from $x_0 = 1$.

(a) Write the equation in the form $f(x) = 0$.

(b) Compute $f'(x)$.

(c) Write the Newton iteration $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ for this problem.

(d) Compute x_1, x_2, x_3 to four decimal places.