Exercises

1. **Derivative Values for 2^x.** Compute each value.

   (a) $\left.\dfrac{d}{dx}(2^x)\right|_{x=1}$.

   (b) $\left.\dfrac{d}{dx}(2^x)\right|_{x=-2}$.

2. **Simplify and Differentiate.** Rewrite each expression in the form $e^{g(x)}$, then differentiate.

   (a) $\left(\dfrac{1}{e^3}\right)^{2x}$.

   (b) e^{1-x^2}.

   (c) e^{3x-1}.

3. **A Tangent Line Parallel to $y = x$.** Find the point on the graph of $f(x) = e^x$ where the tangent line is parallel to $y = x$. Then write the equation of that tangent line.

4. **Chain, Product, and Quotient Rules with Exponentials.** Differentiate each function and simplify.

   (a) $y = (e^{2x} + e^{-2x})^3$.

   (b) $y = \dfrac{x + e^{2x}}{e^x}$.

   (c) $f(x) = \dfrac{3 - 4x}{e^{2x}}$.

5. **An Exponential Equation.** Solve

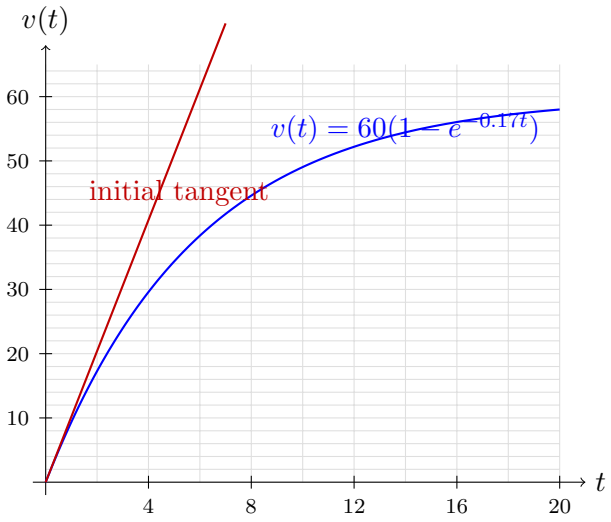
   $$e^{x^2-2x} = e^8.$$

   Explain exactly where injectivity is used.

6. **Parachutist Velocity.** The velocity of a parachutist during free fall is

   $$v(t) = 60(1 - e^{-0.17t})$$

   metres per second. The graph below shows $v(t)$ for $0 \le t \le 20$.



Recall that acceleration is the derivative of velocity.

(a) Estimate the velocity when $t = 8$ seconds.

(b) Compute the acceleration when $t = 0$.

(c) Find exactly when the velocity is 30 metres per second.

(d) Find exactly when the acceleration is 5 metres per second squared.

7. A Two-Reading Exponential Model. A quantity $Q(t)$ satisfies $Q'(t) = kQ(t)$. Suppose

$$Q(2) = 5, \qquad Q'(2) = 3.$$

(a) Find k.

(b) Find a closed form for $Q(t)$.

(c) Find the time at which $Q(t) = 20$.

8. Chain Rule with an Unknown Function. Let f be differentiable, and define

$$F(x) = f(e^{x^2}) + e^{f(x)}.$$

Find $F'(x)$ in terms of f and f'.

9. Recovering a Constant from a Differential Equation. Suppose $A(t) = Ce^{kt}$, where $C > 0$, and suppose

$$A(0) = 12, \qquad A(5) = 27.$$

(a) Find C and k.

(b) Compute $A'(5)$ exactly.

(c) Find the doubling time of A.