1 Exercises

1. **Product Rule Warm-Up.** Differentiate each function. Where it is useful, leave the answer in a factored form first and then expand.

   (a) $y = (x+1)(x^3+5x+2)$.
   (b) $y = x(x^2+1)^4$.
   (c) $y = (5x+1)(x^2-1) + \dfrac{2x+1}{3}$.

2. **Quotient Rule Warm-Up.** Differentiate each function and state the values of x excluded from the natural domain.

   (a) $y = \dfrac{x^2-1}{x^2+1}$.
   (b) $y = \dfrac{x+3}{(2x+1)^2}$.
   (c) $y = \dfrac{1}{\pi} + \dfrac{2}{x^2+1}$.

3. **Horizontal Tangents on a Rational Curve.** Find all x-coordinates of points (x, y) on the curve

   $$y = \frac{(x-2)^5}{(x-4)^3}$$

   where the tangent line is horizontal.

4. **Prescribed Slope on a Rational Curve.** Find the point or points on the graph of

   $$y = \frac{x^2+3x-1}{x}$$

   where the slope is 5.

5. **Rate of Change of Area.** The width of a rectangle is increasing at a rate of 3 inches per second, and its length is increasing at a rate of 4 inches per second. At what rate is the area of the rectangle increasing when its width is 5 inches and its length is 6 inches?

   Hint. Let $W(t)$ and $L(t)$ be the width and length, respectively, at time t, and write $A(t) = W(t)L(t)$.

6. **Computing a Composition.** Let

   $$f(x) = x(x^2+1), \qquad g(x) = \sqrt{x}.$$

   Compute $f(g(x))$ and simplify.

7. **Recognizing Composite Functions.** Each function below may be viewed as a composite function $h(x) = f(g(x))$. Find one possible choice of outside function f and inside function g.

   (a) $h(x) = (x^3+8x-2)^5$.
   (b) $h(x) = \dfrac{1}{x^3-5x^2+1}$.

8. Several Rules at Once I. Differentiate each function using one or more of the differentiation rules discussed thus far.

(a) $y = 6x^2(x-1)^3$.
(b) $y = 2(x^3-1)(3x^2+1)^4$.
(c) $y = \left(\dfrac{3x+1}{4x-1}\right)^3$.

9. Derivative of a Composition. Let

$$f(x) = x^5, \qquad g(x) = 6x - 1.$$

Compute $\dfrac{d}{dx} f(g(x))$.

10. Horizontal Tangents after the Chain Rule. Find the x-coordinates of all points on the curve

$$y = (-x^2 + 4x - 3)^3$$

where the tangent line is horizontal.

11. Marginal Profit and Time Rate of Change. When a company produces and sells x thousand units per week, its total weekly profit is P thousand dollars, where

$$P = \frac{200x}{100 + x^2}.$$

The production level t weeks from the present is

$$x = 4 + 2t.$$

(a) Find the marginal profit $\dfrac{dP}{dx}$.
(b) Find the time rate of change of profit $\dfrac{dP}{dt}$.
(c) How fast, with respect to time, are profits changing when $t = 8$?

12. Recovering the Inside Function. If f and g are differentiable functions and

$$\frac{d}{dx} f(g(x)) = 3x^2\, f'(x^3 + 1),$$

find a possible function $g(x)$.