# Intro to Logic
# (Informal Logic)

Gudfit

# Contents

# 1 Introduction

## 1.1 Basic Concepts

### 1.1.1 Argument, Premises, Conclusions

**Definition 1.1.** (Logic). Logic is the study of methods for evaluating whether the premises of an argument adequately support its conclusion.

Its aim is to develop a set of rules that help us not only evaluate others' arguments but also construct better ones of our own.

**Definition 1.2.** (Argument). An argument is a group of statements (one or more) in which the premises are offered as support for another statement, the conclusion.

**Remark.** Not every passage is an argument. Some are just *unsupported assertions* because they offer no premises for a conclusion. Common non-arguments include:

(i) Reports: Sets of statements intended to provide information.
(ii) Illustrations: Statements accompanied by explanatory examples.
(iii) Explanations: Statements that give reasons why some fact holds.
(iv) Conditionals: If–then statements standing alone (we examine them later).

Arguments come in two broad flavors:

- Deductive: The premises are intended to *guarantee* the conclusion.
- Inductive: The premises are intended to make the conclusion *probable* (no guarantee).

**Example 1.1.**

Deductive: All good dogs go to heaven. My dog is a good dog. Therefore, my dog goes to heaven.

Inductive: In a large, representative survey, *most* artists love math. Daniel is an artist.
Therefore, Daniel *likely* loves math.

Intending to guarantee a conclusion is one thing; actually doing it is another. So deductive arguments can be either *valid* or *invalid*.

- Valid: A deductive argument where, if the premises are true, the conclusion *cannot* be false.
- Invalid: A deductive argument where it is still possible for the premises to be true and the conclusion false.

**Note.** A valid argument is one in which the premises *cannot all be true while the conclusion is false*. If the premises are true, the truth of the conclusion is **guaranteed**.

**Example 1.2.**

All birds have beaks. Some cats are birds. Therefore, some cats have beaks.

The second premise is false in fact, but the *argument is still valid*: if both premises were true, the conclusion would have to be true as well.

**Example 1.3.**

> Some Americans work in education. Donald Knuth is an American.
>
> Therefore, Donald Knuth works in education.

Even if both premises are true, the conclusion need not be; from "Some $B$ are $C$" and "$A$ is $B$" it does not follow that "$A$ is $C$." The premises are compatible with Knuth's being an American who does *not* work in education, so the argument is *invalid*.

**Note.** Whether the premises are actually true is a separate question from whether the argument is valid.

Suppose a valid argument has a false conclusion. Then at least one premise must be false—hence the slogan: **validity preserves truth**. Does validity also preserve falsehood? No:

**Example 1.4.**

> All mammals lay eggs. Bats are mammals. Therefore, bats lay eggs.

This argument is valid in *form* (universal + instance $\Rightarrow$ instance), but here it has a false premise and a false conclusion. So validity does *not* preserve falsity: a false conclusion can still arise when some premise is false. What validity forbids is the combination of *true* premises with a *false* conclusion.

**Example 1.5.**

> All reptiles are mammals. All mammals are vertebrates. Therefore, all reptiles are vertebrates.

This is a valid pattern too. In this instance the first premise is false while the conclusion is true. Again, validity does not preserve falsity.

**Note.** An argument can be valid or invalid regardless of whether we currently know the truth values of its premises or conclusion.

**Definition 1.3.** (Statement). A sentence that is either true or false.

**Example 1.6.** These are *not* statements:

(i) How many apples do you want?
(ii) Let's get some apples.
(iii) Get some oranges!

($i$) is a question (not true/false), ($ii$) a proposal, and ($iii$) a command. None has a truth value.

**Note.** Typically, statements are declarative sentences; many other sentence types are neither true nor false.

**Definition 1.4.** (Premises). Statements that set forth reasons or evidence.

**Definition 1.5.** (Conclusion). The statement the premises are intended to support.

**Example 1.7.**

> All fruit are good for you. An apple is a fruit. Therefore, an apple is good for you.

The first two statements are premises; the third is the conclusion.

**Remark.** Here "therefore" flags the conclusion. The premises really do support it; the argument is valid.

**Example 1.8.**

> Some fruit have a lot of sugar. A lime is a fruit. Therefore, a lime has a lot of sugar.

The premises do *not* guarantee the conclusion, so this is *invalid* as a deductive argument.

**Note.** This is best viewed as a weak inference from an existential claim ("some") to a random instance—not a textbook "hasty generalization," which infers a universal conclusion ("all").

A core skill is distinguishing premises from conclusions. If you swap them, your analysis falls apart. Indicator words help:

| | | | |
|---|---|---|---|
| therefore | hence | consequently | thus |
| accordingly | as a result | it follows that | so |
| entails that | we may conclude | proves that | implies that |
| wherefore | demonstrates that | shows that | indicates that |

Whenever a statement follows one of these, it is usually the conclusion; the others play the role of premises.

**Example 1.9.**

> Cars produce carbon emissions,
> therefore driving cars harms the environment.

"Therefore" functions as a conclusion indicator, while the preceding statement serves as the premise. Additional premise indicators include:

| | | | |
|---|---|---|---|
| since | because | as | for |
| given that | seeing that | as indicated by | due to |
| in that | inasmuch as | on account of | considering that |
| may be inferred from | for the reason that | follows from | granted that |

**Example 1.10.**

> Since apples are high in fiber and bananas contain potassium,
> eating either fruit is good for your health.

"Since" serves as a premise indicator, while the final sentence presents the conclusion. In some cases, one indicator can introduce multiple premises.

**Example 1.11.**

> Since global warming poses an existential threat to humanity, and
>
> industrial pollution accelerates climate change,
>
> immediate environmental action is necessary.

Here "Since" introduces two premises leading to the conclusion.

### 1.1.2 Identifying Arguments

Real-life arguments are rarely tidy. Conclusions can come first, last, or in the middle. Long arguments can be chains of shorter ones. Authors add fluff that does zero logical work. A version that makes the logical features explicit is a *well-crafted argument*.

**Definition 1.6.** (Well-crafted Argument). A presentation in which the logical structure is made explicit.

When indicators are missing, ask:

(i) What is supposed to follow from what?
(ii) What is the arguer trying to prove?
(iii) What ties the other statements together?
(iv) Which statement would the others naturally support?

**Example 1.12.**

> College education has become unaffordable for most Americans.
>
> Student loan debt is about $1.7 trillion.
>
> Average tuition has increased by over 200% in twenty years.
>
> Most graduates spend decades repaying loans.
>
> Wages have not kept pace with education costs.

Here, the first sentence is the conclusion, and the rest are premises. Standardized, the argument reads:

P1: Debt is about $1.7T.
P2: Tuition up 200% in twenty years.
P3: Graduates repay for decades.
P4: Wages have lagged.

Therefore, college has become unaffordable for most Americans.

Some passages include *excess verbiage*: words or sentences that add no support. Strip them out.

**Definition 1.7.** (Excess verbiage). Words or sentences that add no evidential support.

**Example 1.13.**

> Your local gym needs more members.
>
> Although it was built in 1985, the gym was recently renovated.
>
> Membership fees are reasonable; the equipment is modern;
>
> the location is convenient; many members feel healthier after joining.
>
> Also, the mascot is a cartoon dumbbell named "Swolebert."

The mascot line is fluff. The working premises are the renovated facility, reasonable fees, modern equipment, convenient location, and reported health benefits. Arguments often leave some pieces implicit; these are *enthymemes*.

**Definition 1.8.** Enthymeme. An argument with an implicit premise or conclusion.

**Example 1.14.** "Macbeth is mortal because he is human."
Implicit premise: "All humans are mortal."

**Remark.** When you standardize an argument:

(i) Stay faithful to the original.
(ii) Identify premises and conclusion.
(iii) Cut excess verbiage.
(iv) Use complete sentences.
(v) Interpret fairly and charitably.
(vi) Don't confuse sub-conclusions with the final conclusion.
(vii) Make obvious implicit premises explicit—charitably.

### 1.1.3 Arguments vs. Explanations

Not every passage that uses "because" is offering an argument. Sometimes it explains why something (already accepted) is so.

**Example 1.15.**

- Explanation: The sidewalk is wet *because* the sprinkler ran all night.
- Argument: The sidewalk is wet; therefore, it probably rained.

**Remark.** A quick test: Are we trying to establish whether the claim is true (argument) or why an accepted claim is true (explanation)?

### 1.1.4 Sound Arguments

Invalidity means that even if the premises are true, the conclusion *could* be false.

**Example 1.16.**

- All Pringles are chips. All Walkers are chips. Hence, all Pringles are Walkers.
- Gudfit has a crush on Rabbit. Therefore, Rabbit has a crush on Gudfit.

In both cases, the premises could be true while the conclusion is false. So:

(i) Validity alone is not the whole story.
(ii) True premises alone aren't enough either.
(iii) Valid arguments can have false conclusions if some premise is false.
(iv) Invalid arguments can luckily have true conclusions.

**Definition 1.9.** (Sound Argument). A valid argument with all true premises.

**Example 1.17.**

All squares are rectangles. All rectangles have four sides. So, all squares have four sides.

(Valid, and—with true premises—sound.)

**Definition 1.10.** (Unsound Argument). An argument that is invalid, or has at least one false premise (or both).

**Note.** Arguments aren't true or false; *statements* are. Arguments are valid/invalid and (when deductive) sound/unsound.

## 1.2 Forms and Validity

Deductive logic studies methods for determining whether an argument is valid.

**Definition 1.11.** (Argument Form). A reusable pattern of reasoning.

**Definition 1.12.** (Modus Ponens). "The way of affirming." Pattern:

If A, then B.

A.

So, B.

Here 'A' and 'B' can stand for any statements. Replacing them uniformly yields a *substitution instance*.

**Definition 1.13.** (Substitution Instance). What you get by replacing the variables in a form with specific statements, *uniformly*.

**Example 1.18.** Let A be "The sun is bright," and B be "It's useful to wear sunglasses."

> If the sun is bright, then it's useful to wear sunglasses.
>
> The sun is bright.
>
> So, it's useful to wear sunglasses.

**Note.** Modus ponens is valid: no instance has true premises and a false conclusion.

**Definition 1.14.** (Valid Argument Form). A form is valid iff *every* substitution instance is a valid argument.

**Definition 1.15.** (Formally Valid Argument). An argument that is valid *in virtue of its form*.

**Note.** Many valid everyday arguments are formally valid, but some arguments are valid for non-formal reasons—e.g., because their conclusions are necessarily true.

**Remark.** Some arguments are valid simply because their conclusions cannot be false (necessary truths).

**Example 1.19.**

> All mathematicians are nerds. So, one plus one is two.

The conclusion is necessarily true, so the argument is (vacuously) valid even though there's no real support from the premise.

**Definition 1.16.** (Vacuous Validity). An argument is vacuously valid when the conclusion is necessarily true regardless of the premises.

**Definition 1.17.** (Conditional Statement). An if–then statement. The "if" part is the *antecedent*; the "then" part is the *consequent*. Conditionals are hypothetical: they say that *if* the antecedent holds, the consequent must hold.

**Note.** The antecedent is just the content after "if" (not the word "if" itself); likewise for the consequent.

**Remark.** The consequent can be true even when the antecedent isn't—conditionals don't assert either part by themselves.

There are many ways to state a conditional:

| | | | |
|---|---|---|---|
| if | assuming that | provided that | on condition that |
| suppose that | in case | when | whenever |
| given that | granted that | presuming that | in the event that |
| only if | unless | as long as | should |

**Note.** "If A, then B" typically says A is *sufficient* for B. "A only if B" says B is *necessary* for A. "A if and only if B" (the *biconditional*) says each is both necessary and sufficient for the other.

**Example 1.20.**

> Gudfit likes cola only if Gudfit has a sweet tooth.
>
> Gudfit has a sweet tooth only if Gudfit likes cola.

These say different things. The first makes *having a sweet tooth* necessary for *liking cola*. The second makes *liking cola* necessary for *having a sweet tooth*. Neither implies the other.

To see the form more easily, rewrite variants into a standard "if–then" phrasing.

### 1.2.1 Famous Valid Forms

**Definition 1.18.** (Modus Tollens). "The way of denying." Pattern:

> If A, then B.
>
> Not B.
>
> So, not A.

**Definition 1.19.** (Negation). To negate a statement is to deny it; the negation is true exactly when the original is false.

**Note.** The second premise denies the consequent; the conclusion denies the antecedent.

**Remark.** The order of premises does not matter:

> If Gudfit eats chocolate, then Gudfit will be happy.
>
> Gudfit is sad.
>
> So, Gudfit did not eat chocolate.

is equivalent (as an argument) to swapping the first two lines.

**Definition 1.20.** (Hypothetical Syllogism).

> If A, then B.
>
> If B, then C.
>
> So, if A, then C.

**Definition 1.21.** (Disjunctive Syllogism). From a disjunction and the denial of one disjunct, infer the other:

> Either A or B.
>
> Not A.
>
> So, B.

or symmetrically with "Not B; so, A."

**Note.** "A or B" can be read inclusively (one or both) or exclusively (exactly one). In formal settings we usually assume the inclusive reading. It is important to note this is *not* the pattern "Either A or B; A; therefore, not B," which can have true premises and a false conclusion (e.g., with two true mathematical facts).

**Definition 1.22.** (Constructive Dilemma).

> Either A or B.
>
> If A, then C.
>
> If B, then D.
>
> So, either C or D.

This form is always valid.

Knowing these forms lets us use the *famous forms method*:

**Definition 1.23.** (Famous Forms Method).

(i) Identify the component statements (use letters as placeholders).
(ii) Rewrite the argument with those placeholders to expose the pattern.
(iii) Compare to the list of famous valid forms. If it matches, the argument is valid.

**Example 1.21.**

$$\overbrace{\text{Gudfit will get a good grade}}^{A}\ \text{if}\ \overbrace{\text{Gudfit learns algorithms}}^{B}.$$

$$\underbrace{\text{Gudfit learns algorithms.}}_{B}\ \text{Therefore,}\ \underbrace{\text{Gudfit will get a good grade.}}_{A}$$

Pattern:

> If B, then A.
>
> B.
>
> So, A.

That's an instance of modus ponens.

**Remark.** You can *uniformly rename* placeholders (e.g., swap the letters A and B throughout) to recognize a pattern more easily. Renaming does *not* change the logic; it's just a relabeling.

## 1.3 Counterexamples and Invalidity

Some arguments take the shape of well-known valid forms but are actually invalid. For example, consider the following two arguments:

**Example 1.22.** .

> If a number $n$ is divisible by 4, then $n$ is even.
>
> $n$ is not divisible by 4.
>
> So, $n$ is not even.

**Example 1.23.** .

If it is raining, then the ground is wet.

The ground is wet.

Therefore, it is raining.

At first glance, Example 1.22 might seem to imitate an instance of modus tollens. In a proper modus tollens argument, the consequent of the conditional is denied, and the conclusion denies the antecedent. However, in this example the antecedent is denied and the conclusion denies the consequent. This misstep is an instance of the fallacy of denying the antecedent, which is represented as:

**Definition 1.24.** (Fallacy of Denying the Antecedent).

If A, then B.

Not A.

So, Not B.

A counterexample helps illustrate the invalidity of this argument form. Consider the number 6: it is even but not divisible by 4. Here, the premises are true, yet the conclusion is false. This is a clear counterexample.

### 1.3.1 The Counterexample Method

**Definition 1.25.** (Counterexample). A counterexample to an argument *form* is a substitution instance where the premises are well-known truths while the conclusion is a well-known falsehood. Such an instance shows that the *form* does not preserve truth and is therefore invalid.

**Remark.** Using a counterexample tests formal validity. Some arguments are valid for non-formal reasons (e.g., a necessarily true conclusion). In such cases, the form may be invalid even though the particular argument is valid.

Example 1.23 is an example of the fallacy of affirming the consequent. This error occurs when one assumes that if "If A, then B" is true and B is true, then A must also be true. Its structure is as follows:

**Definition 1.26.** (Fallacy of Affirming the Consequent).

If A, then B.

B.

So, A.

While the premises in Example 1.23 might be true, the conclusion does not necessarily follow because there could be other reasons for the ground being wet (for example, a sprinkler). Both examples illustrate invalid argument forms.

**Definition 1.27.** (Invalid Argument Form). An argument form that, under some substitution instances, fails to preserve truth.

**Remark.** This is where mathematics shines: its absolute truths ensure that counterexamples are effective. In contrast to other fields—where premises and conclusions may be less clear-cut—mathematics provides well-known truths that make counterexamples particularly compelling.

**Definition 1.28.** (Good Counterexample). A good counterexample to an argument form is a substitution instance where the premises are well-known truths and the conclusion is a well-known falsehood.

Counterexamples can be used not only to test the invalidity of argument forms but also to identify invalid arguments.

**Example 1.24.** Suppose an argument:

> Cheesecake is mid, and too much cheese cake is good for you.
>
> Cheese is good has alot of fat.
>
> So, Cheese is good for you.

stripping this argument down to its form:

> A and B.
>
> C.
>
> So, D.

One can substitute well-known truths for the premises and a well-known falsehood for the conclusion:

> The Earth revolves around the Sun and Water is composed of hydrogen and oxygen.
>
> 2+2=4
>
> Cheese smells like paint.

To show that the argument is invalid. The reasoning is based on the assumption that if an argument is an instance of an invalid argument form, then the argument itself is invalid. Thus, using the counterexample above, it is shown that this argument is invalid since the instance provided is invalid. Thus using the counter example above it is shown that this argument is invalid as the instance above is invalid.

**Note.** Although this assumption is false, it is useful for identifying invalid arguments.

**Remark.** As shown above, a counterexample need not be good. It only needs to instantiate the form in question, with premises as well-known truths and the conclusion as a well-known falsehood, to serve as a counterexample.

**Example 1.25.** Suppose you have the form:

If A, then B.

If A, then C.

So, if B, then C.

At first glance, this might seem acceptable. Now, consider an instance where the conclusion is quite absurd:

If A, then Gudfit smells like cheese.

If A, then he is a rat.

So, if Gudfit smells like cheese, then he is a rat.

To form a counterexample, replace A with a well-known (or humorously assumed) truth. For instance, let A be "Gudfit doesn't shower." Then the argument becomes:

If Gudfit doesn't shower, then Gudfit smells like cheese.

If Gudfit doesn't shower, then he is a rat.

So, if Gudfit smells like cheese, then he is a rat.

Here the premises are well-known truths and the conclusion is a well-known falsehood (or is Gudfit a rat? Hehe). Either way, because assuming that an argument is invalid if it is an instance of an invalid argument form, this is an invalid argument.

### 1.3.2 Categorical Statements and Arguments

While the counterexample method works well for propositional arguments, a complication arises with arguments that involve categorical statements.

**Definition 1.29.** (Categorical Statement). A categorical statement is one that relates two collections of objects, often signaled by terms like "all," "some," or "no" to indicate whether the statement refers to all, some, or none of the members of a collection.

In such arguments, the premises and conclusions refer to entire collection or groups of objects rather than specific, well-known truths or falsehoods. This broader scope can make it more challenging to construct clear counterexamples, as the truth of categorical statements often depends on additional background assumptions about the collections involved.

**Example 1.26.**

A.

B.

So, C.

This argument form is obviously invalid by counterexample. However, consider the following argument, which is obviously valid (and formally valid), even though a naive counterexample might suggest otherwise:

All $\mathbb{N}$ (natural) numbers are $\mathbb{Q}$ (rational) numbers.

All $\mathbb{Q}$ are $\mathbb{R}$ (real) numbers.

So, $\mathbb{N}$ are $\mathbb{R}$.

To resolve this apparent discrepancy, variables can be used to represent terms as well as statements.

**Definition 1.30.** (Term). A term is a word or phrase that *denotes* a collection of objects or things.

Notice that terms have already been using been in play: $\mathbb{N}$ is a term for all natural numbers, $\mathbb{Q}$ for all rational numbers, and $\mathbb{R}$ for all real numbers. This allows us to rewrite the above argument form as:

All A are B.

All B are C.

So, all A are C.

Regardless of what A, B, and C represent, if B contains A and C contains B, then everything in A is contained in C. This shows that the substitution instance used in the earlier counterexample— while valid for many argument forms—does not work in the case of categorical statements.

**Note.** In other words, our assumption that an argument is invalid merely because it is a substitution instance of an invalid form breaks down here. This is because our previous model, which relied on substitution instances involving well-known truths and falsehoods, labeled categorical statements as invalid.

Thus when showing an argument is invalid by counterexample, one must consider the role of logical keywords. The argument's form is identified using these keywords, and the counterexample must directly demonstrate invalidity—not rely on an assumed invalid substitution.

From this we have other valid argument forms:

All A are B.

Some C are not B.

So, Some C are not A.

**Note.** Obviously if all members of A are (in) B and some members of C are not (in) B, then some C are not (in) A.

All A are B.

Some A are C.

So, Some B are C.

**Note.** Since every A is (in) B, any A that is C must also be (in) B. Therefore, there exists at least one B that is (in) C, which is exactly the conclusion. Of course, not all arguments involving categorical statements are valid.

**Example 1.27.**

All A are B.

Some B are not C.

So, Some A are not C.

which is obviously invalid. (The conclusion would only be valid if at least one element of A were guaranteed to fall into the portion of B that is not C).

All cats are animals.

Some animals are not mammals.

So, Some cats are not mammals.

**Note.** For categorical arguments (e.g., "All A are B"), counterexamples often work best by diagramming the sets in mind or by picking natural categories (e.g., cats, mammals, reptiles) so that "true-premise/false-conclusion" cases are transparent.

### 1.3.3 TLDR

Thus for easy reference the counterexample method can be summarized as follows:

**Definition 1.31.** (Counterexample Method).

(i) Extract the argument's form (use uniform placeholders; keep quantifier/copula words when categorical).
(ii) Substitute so the premises are well-known truths.
(iii) Make the conclusion-substitute a well-known falsehood.
(iv) If you get true premises and a false conclusion, the form is invalid; any argument that relies on that form is *formally* invalid.

## 1.4 Strength and Cogency

The focus so far has been on deductive arguments, which aim to guarantee the truth of the conclusion with a valid argument. In contrast, the goal of an inductive argument is to make the conclusion probable rather than certain; it succeeds if the premises being true makes the conclusion likely, though not necessarily true.

**Definition 1.32.** (Strong argument). One in which, if the premises are true, the conclusion is *probable*.

**Definition 1.33.** (Weak argument). One in which, even if the premises are true, the conclusion is *not* made probable.

**Note.** An argument can be strong but not valid.

**Example 1.28.**

- Strong Argument: Gudfit has a 99% failure rate when asking out his crushes. Gudfit asked out Rabbit, so it is highly likely that Gudfit got rejected.
- Weak Argument: Although only 2% of students fail math due to rare edge cases, Gudfit is taking math. This low probability does not provide strong support for the claim that Gudfit will fail math.

The distinction between strong and weak inductive arguments can also be illustrated through various types of inductive reasoning beyond statistical syllogisms. Other forms include:

(i) Arguments from Authority: These arguments rely on the credibility or expertise of a source. A claim supported by a highly authoritative source is considered strong, whereas one supported by a less authoritative source is considered weak.

(ii) Arguments from Analogy: These arguments compare two situations or objects that share relevant similarities. If the similarities are pertinent to the claim being made, the argument is strong; if the similarities are superficial or irrelevant, the argument is weak.

Each type of argument should be evaluated based on the relevance and strength of the evidence provided by the source or the analogy.

**Remark.** All of these examples show that unlike validity, which is all-or-nothing, inductive strength comes in degrees—one strong argument can be stronger than another. By contrast, it makes no sense to call one valid argument "more valid" than another.

**Note.** Because the degree of strength matters in inductive arguments, the best argument is, of course, the strongest one. All else being equal, an inductive argument is better if all its premises are true.

**Definition 1.34.** (Cogent argument). A *strong* inductive argument with all true premises.

**Note.** A cogent argument may still have a false conclusion (its premises don't guarantee it).

**Definition 1.35.** (Uncogent argument). An inductive argument that is *weak*, or has at least one false premise, or both.

## 2 Logic and Language

Earlier it was said that arguments are built from statements. Logicians, however, prefer to say that arguments are built from propositions.

**Definition 2.1.** (Proposition). A truth or falsehood that may or may not be expressed in a sentence.

**Note.** A proposition represents the abstract content or meaning of a statement. Because the same idea can be expressed in various ways, different sentences can convey the same proposition. Conversely, a single sentence may be ambiguous or compound and thus express more than one proposition.

**Remark.** Words and phrases change over time due to cultural shifts and new research.

**Example 2.1.** Suppose one tried to redefine "square" as a "curved figure." Before this redefinition, the proposition "there are square circles" would always be false. However, after the change, one could state "there exist some square circles." But notice that this is a different proposition.

This is why logicians prefer propositions because altering the words only changes the expression, not the truth value, of a proposition. Logic focuses on truth, not wordplay; synonym substitution preserves an argument's structure.

Statements often carry both emotional force and cognitive meaning. However, logic primarily focuses on cognitive meaning—that is, the logical connections between a statement's informational content—because emotional force can interfere with logical insight.

**Definition 2.2.** (Cognitive). The cognitive meaning of a sentence refers to its truth-conditional aspects, which can be analyzed and evaluated for accuracy or validity.

**Definition 2.3.** (Emotive force). The emotive force of a sentence refers to its ability to evoke feelings, attitudes, or reactions independent of its factual content.

Emotive force can disrupt logical insight in at least two ways:

(i) Loaded language, which hinders a clear grasp of the statement's informational context.
(ii) Emotionally charged language, which may discourage seeking or considering further evidence.

## 2.1 Definitions

Another problem with statements is the use of vague or ambiguous words, where definitions, logic, and conclusions are not clearly defined.

Definitions play an important role in an argument because they can make vague ideas more precise. To gain clarity, it is essential to distinguish between the extension and intension of terms.

**Definition 2.4.** (Extension). The extension of a term is the collection of all objects or instances to which the term applies.

**Definition 2.5.** (Intension). The intension of a term is its inherent meaning, including the properties and attributes that determine its applicability.

"There are many ways of specifying the meanings of words; consequently, there are many different types of definitions. To begin with, we may specify the meaning of a word through its extension, or we may specify its meaning through its intension. There is thus a basic distinction between extensional definition and intensional definitions." - Wesley C. Salmon

**Definition 2.6.** (Extensional definition). An extensional definition specifies the meaning of a term by indicating the collection of things to which the term applies. It comes in two basic types:

(i) Non-Verbal: Pointing to paradigm cases (e.g., showing several fish to teach "fish"). Only a sample can be shown.
(ii) Verbal: Listing the members of the extension either *individually* or by *subclasses*.

**Definition 2.7.** (Enumerative definition). An enumerative definition names the members of the extension *individually*.

**Definition 2.8.** (Definition by subclass). Definition by subclass names the members of the extension in *groups*.

**Remark.** Regardless of whether the extension of a word is provided verbally or nonverbally, it is usually impractical or impossible to indicate every member of the extension (especially since some terms cannot be defined extensionally if their extensions are empty). This limitation highlights the need for intensional definitions.

**Definition 2.9.** (Intensional definition). An intensional definition specifies the meaning of a term by indicating the properties a thing must have to be included in the term's extension.

**Note.** Intensional definitions are verbal and come in four main types:

(i) Lexical: Reports the conventional or established definitions of a term.
(ii) Stipulative: Specifies the intension of a term independently of conventional or established use.
(iii) Precising: Reduces the vagueness of a term by imposing limits on its conventional meaning.
(iv) Theoretical: Attempts to provide an adequate explanation by relating a term to a broader theoretical framework.

One technique for constructing definitions that eliminates ambiguity and vagueness is "genus and difference." This method works by defining the definiendum and its definiens.

**Definition 2.10.** (Definiendum and Definiens). The definiendum is the word being defined, and the definens is the word or words used in the definition.

Secondly, the method involves specifying a proper subclass. A class $X$ is a subclass of another class $Y$ if every member of $X$ is also a member of $Y$. The term "proper" indicates that while $X$ is a subclass of $Y$, $Y$ contains members that are not in $X$.

**Definition 2.11.** (Genus). The genus is the general class or category to which the definiendum belongs.

**Definition 2.12.** (Difference). The difference is the specific attribute that distinguishes the definiendum from other members of its genus.

Thus, by specifying both the genus and the difference, a definition clearly identifies the general category of the term and highlights the unique characteristic that sets it apart from other members of that category.

**Remark.** A caution: avoid *circular definitions*, where a term is defined directly or indirectly in terms of itself. Such definitions fail to provide informative or clarifying information and can lead to confusion.

**Note.** It is also important to consider the role of context and pragmatics. While definitions aim to capture the core meaning (or intension) and scope (or extension) of terms, real-world usage can influence how terms are understood. The context in which a term is used may affect its interpretation, adding subtle layers beyond the formal definition.

Definition by genus and difference must conform to the following six criteria:

(i) The definition should not be too broad (i.e., include too many things).
(ii) The definition should not be too narrow (i.e., include too few things).
(iii) The definition should be clear and not ambiguous or figurative.
(iv) The definition should not be circular.
(v) The definition should be affirmative rather than negative, if possible.
(vi) The definition should not select its extension based on attributes that are unsuitable given the context or purpose.

**Remark.** Additionally, it is worth noting the distinction between analytic and synthetic statements. Analytic statements are true by virtue of their definitions (or are true by definition), while synthetic statements require empirical verification. This distinction further emphasizes the foundational role of clear definitions in logical analysis.

If we aren't careful with language, two problems are likely to occur:

(i) Equivocation: A key word or phrase is used with more than one meaning within the argument, though validity requires a consistent meaning.
(ii) Verbal Dispute: The parties *appear to disagree*, but the clash is merely about words; once meanings are clarified, there may be no substantive disagreement.

Some people mistakenly conflate verbal dispute with a persuasive definition.

**Definition 2.13.** (Persuasive definition). A persuasive definition is a definition that is biased in favor of a certain conclusion.

By defining terms in a particular way, one can influence the interpretation of an argument to support a predetermined outcome.

### 2.1.1 Rhetorical Devices (Brief Guide)

Rhetorical coloring can sway judgment without adding evidence.

**Definition 2.14.** (Euphemism / Dysphemism). Soften or harden emotional tone ("collateral damage" vs "civilian deaths").

**Definition 2.15.** (Weasel Words). Vague hedges that mimic evidence ("studies suggest…", "many experts say…" with no citation).

**Definition 2.16.** (Proof Surrogates). Implying support without giving it ("it's well known that…").

**Definition 2.17.** (Framing). Presenting equivalent information in different ways to trigger different reactions ("90% fat-free" vs "10% fat").

**Note.** Spotting rhetorical devices helps separate emotive force from cognitive content.

## 3 Informal Fallacies

Some errors in reasoning tend to be psychologically persuasive. These errors are called fallacies, which can be split into two main types:

**Definition 3.1.** (Formal and Informal Fallacy). A formal fallacy is an error in reasoning that involves the explicit use of an invalid logical form, whereas an informal fallacy is an error that does not involve a clearly invalid form but still undermines the argument.

Informal fallacies are particularly challenging because they require careful examination of an argument's context. Often, they involve premises that are logically irrelevant to the conclusion but may seem relevant due to psychological factors. Examples include:

(i) Ad Hominem Fallacy: Instead of addressing the argument, the premises attack the person making the argument, implying that the argument is false or unsound.
(ii) Strawman: The premises misrepresent or distort the opposing view, making it easier to refute, which leads to an erroneous conclusion.
(iii) Ad Baculum: This fallacy appeals to force or the threat of force rather than providing logical reasoning.
(iv) Ad Populum: The argument appeals to the popularity of a claim as evidence for its truth.
(v) Ad Misericordiam: The argument appeals to pity or emotion instead of presenting a logical case.
(vi) Ad Ignorantiam: The argument claims that a proposition is true simply because it has not been proven false, or false because it has not been proven true.
(vii) Red Herring: The argument diverts attention from the original issue by introducing irrelevant information.

And other times, arguments are flawed because they contain ambiguous words or statements—a category of fallacies involving ambiguity. Aside from equivocation, there are three others:

(i) Amphiboly: Ambiguity from sentence structure yields multiple readings.
(ii) Composition: Illicitly inferring that because the *parts* (or members) have a property, the *whole* (or group) has it.
(iii) Division: Illicitly inferring that because the *whole* (or group) has a property, the *parts* (or members) have it.

And finally, some errors in reasoning occur when an argument makes an unwarranted assumption.

**Definition 3.2.** (Unwarranted Assumption). An assumption that, within the context of an argument, requires support. When this support is not provided, the assumption remains unjustified, thereby undermining the entire argument.

Examples of such errors include:

(i) Petitio Principii (Begging the Question): This fallacy occurs when an argument assumes its conclusion within its premises, resulting in circular reasoning that offers no independent support for the conclusion.
(ii) False Dilemma:This fallacy presents only two alternatives as if they are the only possibilities, when in fact other options may exist. It forces a choice between extremes without considering intermediate possibilities.

(iii) Ad Verecundiam (Appeal to Authority): This fallacy relies on the opinion of an authority figure as evidence for a claim, even when that authority may not be qualified in the relevant area or the issue is not within their expertise.
(iv) False Cause: This fallacy assumes a causal relationship between two events without sufficient evidence to support that connection, confusing correlation with causation.
(v) Fallacy of Complex Question: This fallacy occurs when a question is framed in such a way that it presupposes an answer, forcing the respondent to accept an unwarranted assumption without proper justification.

In each of these cases, the lack of adequate support for key assumptions significantly weakens the argument, rendering it unsound or invalid.

## 3.1 Generalization and Evidence Pitfalls

**Definition 3.3.** (Hasty Generalization). Drawing a broad conclusion from too few or unrepresentative cases.

**Definition 3.4.** (Biased Sample). Sampling from a skewed subpopulation, then projecting to the whole.

**Definition 3.5.** (Anecdotal Evidence). Relying on striking stories instead of adequate data.

**Definition 3.6.** (Suppressed Evidence). Omitting known, relevant information that would weaken the conclusion.

**Definition 3.7.** (Slippery Slope). Claiming (without adequate support) that a small step will trigger a disastrous chain.

**Remark.** When generalizing, look for adequate sample size, representativeness, and absence of overlooked counterevidence.