Exercises

1. Three Points Determine a Plane. Consider the three different points

$$A = (3, -2, 5), \qquad B = \left(\tfrac{1}{2}, 0, 4\right), \qquad C = (1, -2, 10)$$

in $\mathbb{R}^3$.

(a) Use difference vectors to show that these points are not collinear.
(b) Deduce that there is exactly one plane containing them, and give a parametric form for that plane.

2. Equation of the Same Plane. Give an equation for the plane found in the previous problem.

3. Parametrising from an Equation. Find a parametric form for the plane in $\mathbb{R}^3$ given by

$$6x - 6y - z = 7.$$

4. A Parallel Plane Through a Point. Find an equation for the plane parallel to

$$4x - 7y + 2z = 1$$

and passing through the point

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

5. Two Intersecting Lines Determine a Plane. Consider the lines ℓ_1 and ℓ_2 in $\mathbb{R}^3$ given by

$$\ell_1 : \begin{bmatrix} -4 \\ 2+2t_1 \\ 7+3t_1 \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \\ 7 \end{bmatrix} + t_1 \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}, \qquad \ell_2 : \begin{bmatrix} 1+5t_2 \\ 0 \\ 5+t_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} + t_2 \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}.$$

(a) Show that these two lines intersect.
(b) Using their common point, give a parametric form for the plane containing both lines.
(c) Give an equation of this plane.

6. Testing Points and Lines Against a Plane. Consider the plane P defined by

$$-x + 5y + 2z = 3,$$

which is also described by the parametric form

$$\begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} + t' \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}.$$

(a) Is the point

$$\begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$$

on the plane P?
(b) Do the points

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

lie on the same side of P?
(c) Give an example of a parametric form of a line contained in P, and verify that all points on your line satisfy the equation of P.

7. Line–Plane Intersection. Let

$$L:\ \mathbf{x}(t) = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$$

and let

$$E:\ 2x + y - z = 5.$$

Determine whether L meets E, and if so find the intersection point.

8. Relative Position of a Line and a Plane. Let

$$E:\ x - 2y + z = 3$$

and

$$L:\ \mathbf{x}(t) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}.$$

Determine whether L lies in E, is parallel to E, or meets E in a unique point. Justify your answer.